# Supersymmetric electrodynamics of charged and neutral fermions in the extended Wheeler-Feynman approach

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(February 1, 2008)

## Abstract

A supersymmetric formulation of the classical action of interacting charged and neutral fermions with arbitrary anomalous magnetic moment is considered. This formulation generalizes the known action for scalar charged particles investigated in papers by Fokker, Schwarzschild, Tetrode, Wheeler and Feynman. The superfield formulation of the electrodynamics of the Maxwell supermultiplet, constructed from the world coordinates of charged or neutral fermions is carried out basing on the proposed action.

The action at a distance approach developed by Fokker-Schwarzschild-Tetrode-Wheeler-Feynman (FSTWF) revealed a deep connection between classical electromagnetic field and world-line coordinates of relativistic charged particles [4]. The fundamental character of the connections between fields and world coordinates was realized in a new light in string theory where the original classical string lagrangian was formulated in terms of string world-sheet coordinates [5]. The unification of the FSTWF approach with string theory performed by Ramond and Kalb resulted in the discovery of antisymmetric gauge field [8]. Today this antisymmetric field plays an important role in supergravity, superstring and modern topological theories. Moreover, the cosmological studies of Hoyle and Narlikar [6] shed a new light on significance of the FSTWF approach for the solving the old problem of the relation between microworld physics and the features of the Universe as a whole.

As is known, the construction of successive field theory of string interaction involves severe difficulties. We hope that FSTWF approach may be useful for the construction of the theory of (super)string interaction. In this connection the question about the possibility of the unification of the AAD principle together with the principle of supersymmetry seems

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the most important question, since supersymmetry is the natural basis for modern string theory construction. By now, we solved this problem in a case of superparticles. The obtained supersymmetrical generalization allows to elaborate the need acting strategy and can form the basis for construction of (super)string interaction.

The FSTWF approach is based on the action of the two interacted charged particles in terms of its world-line coordinates $x^\mu(t)$ and $y^\mu(\tau)$

$$S_{FSTWF} = e_1 e_2 \int dt \int d\tau\, \dot{x}^\mu \dot{y}_\mu \delta(s_0^2). \tag{1}$$

Here $s_0^\mu$ is the relativistic interval and $\delta$ is the Dirac-function. The variation of this action together with the ordinary free action gives the Lorentz equation of motion. The strength of classical electromagnetic field

$$a^\mu(x) = e \int d\tau \dot{y}^\mu(\tau) \delta(s_0^2) \tag{2}$$

appears in the right hand side of this equation. This field satisfies the Maxwell equation with the current, which is given in a standard way

$$\partial^\mu f_{\mu\nu}(x) = -4\pi j_\nu(x), \qquad j^\mu(x) = e \int d\tau \dot{y}^\mu \delta^{(4)}(s_0) \tag{3}$$

and the Lorentz gauge condition

$$\partial^\mu a_\mu(x) = 0. \tag{4}$$

In order to supersymmetrize the electromagnetic potential (2) and the current we extend the original Minkowski space to superspace. $z^M = (x^\mu(t), \theta^\alpha(t), \bar{\theta}_{\dot{\alpha}}(t))$ and $\zeta^M = (y^\mu(\tau), \xi^\alpha(\tau), \bar{\xi}_{\dot{\alpha}}(\tau))$ are the superspace coordinates of particles, where additional Grassmann coordinates $\theta$ and $\xi$ are the Weyl spinors. The supersymmetry transformations mix the ordinary vector and spinor coordinates of particles [9]

$$\begin{aligned} \delta x^\mu &= i\theta\sigma^\mu\bar{\epsilon} - i\epsilon\sigma^\mu\bar{\theta}, \quad \delta\theta^\alpha = \epsilon^\alpha,\ \delta\bar{\theta}_{\dot{\alpha}} = \bar{\epsilon}_{\dot{\alpha}} \\ \delta y^\mu &= i\xi\sigma^\mu\bar{\epsilon} - i\epsilon\sigma^\mu\bar{\xi}, \quad \delta\xi^\alpha = \epsilon^\alpha,\ \delta\bar{\xi}_{\dot{\alpha}} = \bar{\epsilon}_{\dot{\alpha}}. \end{aligned} \tag{5}$$

The simplest generalizations of the interval $s_0^\mu$ and the velocity $\dot{y}^\mu(\tau)$ invariant under the global supersymmetry transformations (5) are

$$\begin{aligned} s^\mu &= x^\mu - y^\mu - i(\theta\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\theta}), \\ \omega^\mu_\tau &= \dot{y}^\mu - i(\dot{\xi}\sigma^\mu\bar{\xi} - \xi\sigma^\mu\dot{\bar{\xi}}). \end{aligned} \tag{6}$$

However, as will be shown further, it is more convenient to work with the basis of chiral coordinates $(x^\mu_L, \theta^\alpha)$, $(x^\mu_R, \bar{\theta}_{\dot{\alpha}})$ [9].

$$\begin{aligned} x^\mu_L &\equiv x^\mu + i\theta\sigma^\mu\bar{\theta}, \quad y^\mu_L \equiv y^\mu + i\xi\sigma^\mu\bar{\xi}, \\ x^\mu_R &\equiv x^\mu - i\theta\sigma^\mu\bar{\theta}, \quad y^\mu_R \equiv y^\mu - i\xi\sigma^\mu\bar{\xi} \end{aligned} \tag{7}$$

Then it is not so difficult to construct the intervals

$$
\begin{aligned}
s_L^\mu &= x_L^\mu - y_R^\mu - 2i\theta\sigma^\mu\bar{\xi} = s^\mu + i\Delta\sigma^\mu\bar{\Delta}, \quad \Delta^\alpha = \theta^\alpha - \xi^\alpha, \\
s_R^\mu &= x_R^\mu - y_L^\mu + 2i\xi\sigma^\mu\bar{\theta} = s^\mu - i\Delta\sigma^\mu\bar{\Delta}, \quad \bar{\Delta}^{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}} - \bar{\xi}^{\dot{\alpha}},
\end{aligned}
\tag{8}
$$

which are invariant under the SUSY transformations (5) and satisfy the chirality conditions

$$
\begin{aligned}
D_\alpha s_R^\mu &= \left(\frac{\partial}{\partial\theta^\alpha} + i(\sigma^\nu\bar{\theta})_\alpha\partial_\nu\right) s_R^\mu = D_\alpha\bar{\Delta}_{\dot{\beta}} = 0, \\
\bar{D}_{\dot{\alpha}} s_L^\mu &= \left(-\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^\nu)_{\dot{\alpha}}\partial_\nu\right) s_L^\mu = \bar{D}_{\dot{\alpha}}\Delta^\beta = 0.
\end{aligned}
\tag{9}
$$

The main objects of the supersymmetric electodynamics are the superfield vector and spinor connections $A^M = (A^\mu, A^\alpha, \bar{A}_{\dot{\alpha}})$. These potentials play the same role as the vector electromagnetic potential in the standard electrodynamics. The corresponding strength components $F_{MN}$ are not independent superfields due to the standard constraints [9]

$$
F_{\alpha\beta} = F_{\dot{\alpha}\dot{\beta}} = F_{\alpha\dot{\beta}} = 0. \tag{10}
$$

Then all nonzero components of the strength $F_{MN}$ can be expressed through the chiral superfields $W_\alpha$ and its complex conjugated ones

$$
W^\alpha \equiv \frac{i}{4}F_{\mu\dot{\alpha}}\tilde{\sigma}^{\mu\dot{\alpha}\alpha}, \quad \bar{W}^{\dot{\alpha}} \equiv \frac{i}{4}\tilde{\sigma}^{\mu\dot{\alpha}\alpha}F_{\mu\alpha}. \tag{11}
$$

Standard constraints are automatically satisfied if the vector superfield $A_\mu$ has the form

$$
A_\mu = -\frac{i}{4}\tilde{\sigma}_\mu^{\dot{\alpha}\alpha}\left(D_\alpha\bar{A}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}}A_\alpha\right) \tag{12}
$$

and the spinor superfields $A^\alpha$ and $\bar{A}_{\dot{\alpha}}$ are chiral

$$
D_\alpha A_\beta = 0, \quad \bar{D}_{\dot{\alpha}}\bar{A}_{\dot{\beta}} = 0. \tag{13}
$$

The above expressions together with the conjugation condition for the spinor superfields allow to reduce the problem of construction of the superpotentials $A^M$ to the problem of obtaining the spinor superfield $A^\alpha$.

Let us seek for an integral representation for spinor potential in the form

$$
A_\alpha(x,\theta,\bar{\theta}) = \int d\tau\mathcal{K}_\alpha(\omega_\tau, \dot{\xi}, \dot{\bar{\xi}}, \bar{\Delta})\delta(s_R^2), \tag{14}
$$

where the kernel $\mathcal{K}_\alpha$ is the chiral supersymmetric invariant operator and has a dimensionality $L^{\frac{1}{2}}$ in system $h = c = 1$.

Then spinor strength $W$ can be rewritten by the following way [9]

$$
\begin{aligned}
W^\alpha = &\frac{1}{2}\int d\tau\Big[\frac{1}{4}\bar{D}\bar{D}(\mathcal{K}^\alpha) - i(\Delta\sigma^\rho\bar{D})(\mathcal{K}^\alpha)\partial_\rho - \mathcal{K}^\alpha(\Delta\Delta)\Box\Big]\delta(s_R^2) \\
&-\frac{i}{2}\int d\tau\mathcal{K}_{\dot{\alpha}}^*\partial^{\dot{\alpha}\alpha}\delta(s_L^2).
\end{aligned}
\tag{15}
$$

We use the superfields $W$ and $\bar{W}$ in order to construct the representation for supersymmetric current

$$-4\pi\mathcal{J} = D^{\alpha}W_{\alpha} + \bar{D}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}. \tag{16}$$

Further we assume that the following condition for kernel $\mathcal{K}$

$$(\bar{D}\sigma^{\mu}\mathcal{K}) \sim \frac{d}{d\tau}(s_R^{\mu}) \tag{17}$$

is satisfied. This condition plays the same role, as the Lorentz gauge condition in the original FSTWF theory — it permits to write the above expression in the form of wave equation

$$\begin{aligned} &\Box\Phi(x,\theta,\bar{\theta}) = -4\pi\mathcal{J}(x,\theta,\bar{\theta}), \text{ where} \\ &\qquad \Phi = -2\int d\tau\left(\Delta^{\alpha}\mathcal{K}_{\alpha}\delta(s_R^2) + \mathcal{K}^{*}_{\dot{\alpha}}\bar{\Delta}^{\dot{\alpha}}\delta(s_L^2)\right). \end{aligned} \tag{18}$$

As one can see, the fundamental property of the interval

$$\Box\delta(s^2) = -4\pi\delta^{(4)}(s^{\nu}) \tag{19}$$

gives the possibility to write an integral representation for the supersymmetric current

$$\mathcal{J} = -4\int d\tau\Delta^{\alpha}\mathcal{K}_{\alpha}\delta^{(4)}(s_R) + \mathcal{K}^{*}_{\dot{\alpha}}\bar{\Delta}^{\dot{\alpha}}\delta^{(4)}(s_L). \tag{20}$$

This wave equation is a superfield generalization of the Maxwell equations in the Lorentz gauge. The superfield $\Phi$ has the physical meaning of the prepotential $V$ evaluated in the superfield gauge

$$\left\{DD, \bar{D}\bar{D}\right\}V = 0 \;\Rightarrow V(x,\theta,\bar{\theta}) = \frac{1}{4}\Phi(x,\theta,\bar{\theta}). \tag{21}$$

The superfield gauge condition (21) is split into the following component gauge conditions

$$\begin{aligned} &\partial^{\dot{\alpha}\alpha}\chi_{\alpha}(x) = i\bar{\lambda}^{\dot{\alpha}}(x),\ \partial^{\dot{\alpha}\alpha}\bar{\chi}_{\dot{\alpha}}(x) = -i\lambda^{\alpha}(x), \\ &\Box C(x) = -D(x),\ M(x) = N(x) = 0, \\ &\partial_{\mu}v^{\mu}(x) = 0. \end{aligned} \tag{22}$$

Here $v^{\mu}(x)$, $\lambda^{\alpha}(x)$ and $D(x)$ — are the Maxwell supermultiplet fields, $\chi$, $M$, $N$, $C$ — are the component fields, which vanish in Wess-Zumino gauge. Above gauge conditions (22) permit to represent prepotential $V(x,\theta,\bar{\theta})$ as the following component decomposition

$$\begin{aligned} V(x,\theta,\bar{\theta}) = &-\Box^{-1}D - \theta^{\alpha}(\partial^{-1})_{\alpha\dot{\beta}}\lambda^{\dot{\beta}} - i\bar{\theta}_{\dot{\alpha}}(\partial^{-1})^{\dot{\alpha}\beta}\lambda_{\beta} - (\theta\sigma_{\rho}\bar{\theta})v^{\rho} \\ &+\frac{i}{2}\theta\theta\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}} - \frac{i}{2}\bar{\theta}\bar{\theta}\theta^{\alpha}\lambda_{\alpha} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}D \end{aligned} \tag{23}$$

with the vector $v^{\mu}(x)$, two spinors $\lambda^{\alpha}(x)$, $\bar{\lambda}_{\dot{\alpha}}(x)$ and an auxiliary scalar field $D(x)$ forming the Maxwell multiplet.

The components of supercurrent can be defined as

$$\mathcal{J} = -4j^{(0)} + 4\theta^\alpha j^{(1)}_\alpha - 4\bar\theta_{\dot\alpha}\bar j^{\dot\alpha(1)} - 4(\theta\sigma_\rho\bar\theta)j^{(2)\rho}$$
$$-2i\theta\theta\bar\theta_{\dot\alpha}\partial^{\dot\alpha\alpha}j^{(1)}_\alpha + 2i\bar\theta\bar\theta\theta^\alpha\partial_{\alpha\dot\alpha}\bar j^{(1)\dot\alpha} + \theta\theta\bar\theta\bar\theta\Box j^{(0)}. \tag{24}$$

After substitution of both of these decompositions into the superfield wave equation one can obtain that the component fields of $\Phi$ satisfy the Maxwell and Dirac equations with currents

$$\Box v^\mu(x) = -4\pi j^{(2)\mu}(x),\ \partial_{\alpha\dot\alpha}\bar\lambda^{\dot\alpha}(x) = -4\pi j^{(1)}_\alpha(x). \tag{25}$$

Now we consider the representations for kernel in terms of the intervals and velocities. These quantities are chiral and supersymmetry invariant ones. Thus the constructed kernel

$$\mathcal{K}^{(e)}_\alpha = e\sigma^\mu_{\alpha\dot\alpha}(\omega_{\tau\mu} + 2i(\bar\Delta\tilde\sigma_\mu\dot\xi))\bar\Delta^{\dot\alpha}, \tag{26}$$

is chiral and supersymmetry invariant operator too. It is not so difficult to verify that this kernel satisfies the above introduced condition for kernel and has the appropriate dimensionality. Note, that the corresponding vector superpotential satisfies the superfield Lorentz gauge condition

$$\partial_\mu A^\mu = 0.$$

If we substitute this representation of kernel $\mathcal{K}$ into expressions for superfield potentials, strengths and current they can be rewritten in terms of superspace coordinates [11]

$$A_\alpha = e\int d\tau(\omega_{\tau\mu}\sigma^\mu_{\alpha\dot\alpha}\bar\Delta^{\dot\alpha} + 2i\dot\xi_\alpha\bar\Delta\bar\Delta)\delta(s^2_R),$$
$$\bar A_{\dot\alpha} = -e\int d\tau(\omega_{\tau\mu}\Delta^\alpha\sigma^\mu_{\alpha\dot\alpha} - 2i\dot{\bar\xi}_{\dot\alpha}\Delta\Delta)\delta(s^2_L),$$
$$A_\mu = -ie\int d\tau\Big[\omega_{\tau\mu} - \varepsilon_{\mu\nu\rho\lambda}\omega^\nu_\tau(\Delta\sigma^\rho\bar\Delta)\partial^\lambda$$
$$+i\Big((\Delta\sigma_\mu\dot{\bar\xi}) - (\dot\xi\sigma_\mu\bar\Delta)\Big) + \frac{1}{4}\Delta\Delta\bar\Delta\bar\Delta\omega^\nu_\tau\Big(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\Big)$$
$$+\Big(\Delta\Delta(\dot{\bar\xi}\tilde\sigma_{\mu\rho}\bar\Delta) + (\dot\xi\sigma_{\mu\rho}\Delta)\bar\Delta\bar\Delta\Big)\,\partial^\rho\Big]\delta(s^2), \tag{27}$$

$$W^\alpha = -ie\int d\tau\Big[\dot\xi^\alpha + i\dot\xi^\alpha\Delta\sigma^\mu\bar\Delta\partial_\mu + \frac{1}{4}\dot\xi^\alpha\Delta\Delta\bar\Delta\bar\Delta\Box$$
$$+\omega_{\tau\mu}\Big(2(\Delta\sigma^{\mu\nu})^\alpha\partial_\nu - \frac{i}{2}\Delta\Delta(\bar\Delta\tilde\sigma_\nu)^\alpha(\partial^\mu\partial^\nu, -\eta^{\mu\nu}\Box)\Big)$$
$$-i\Delta\Delta(\dot{\bar\xi}\tilde\sigma_\mu)^\alpha\partial^\mu\Big]\delta(s^2),$$
$$\Phi = -2e\int d\tau\Big(\omega^\mu_\tau(\Delta\sigma_\mu\bar\Delta) + i(\dot\xi\Delta)\bar\Delta\bar\Delta - i\Delta\Delta(\dot{\bar\xi}\bar\Delta)\Big)\delta(s^2). \tag{28}$$

The integral representations of the component fields in terms of the superspace coordinates can be written as zero-terms of the corresponding superfields

$$v_\mu(x) = iA_\mu\Big|_{\theta=0} = e\int d\tau\Big[\dot y_\mu - \varepsilon_{\mu\nu\rho\lambda}\dot y^\nu(\xi\sigma^\rho\bar\xi)\partial^\lambda$$
$$+\Big(\xi\xi(\dot{\bar\xi}\tilde\sigma_{\mu\rho}\bar\xi) + (\dot\xi\sigma_{\mu\rho}\xi)\bar\xi\bar\xi\Big)\,\partial^\rho + \frac{1}{4}\xi\xi\bar\xi\bar\xi\dot y^\nu\left(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\right)\Big]\delta(s^2_0),$$

$$\lambda^\alpha(x) = iW_\alpha\Big|_{\theta=0} = e\int d\tau\Big[\dot{\xi}^\alpha - i\dot{\xi}\xi(\bar{\xi}\tilde{\sigma}_\mu)^\alpha\partial^\mu + \frac{i}{2}\xi\xi(\dot{\bar{\xi}}\tilde{\sigma}_\mu)^\alpha\partial^\mu$$
$$-\frac{1}{2}\dot{\xi}^\alpha\xi\xi\bar{\xi}\bar{\xi}\Box + \dot{y}_\mu\Big(-2(\xi\sigma^{\mu\nu})^\alpha\partial_\nu + \frac{i}{2}\xi\xi(\bar{\xi}\tilde{\sigma}_\nu)^\alpha(\partial^\mu\partial^\nu - \eta^{\mu\nu}\Box)\Big)\Big]\delta(s_0^2),$$
$$D(x) = -\frac{1}{4}\Box\Phi\Big|_{\theta=0} = e\int d\tau\Big[\dot{y}_\mu(\xi\sigma^\mu\bar{\xi}) - i\left(\xi\xi(\dot{\bar{\xi}}\bar{\xi}) - (\dot{\xi}\xi)\bar{\xi}\bar{\xi}\right)\Big]\Box\delta(s_0^2). \tag{29}$$

It is obvious, that the electromagnetic potential $v^\mu(x)$ is the desired supersymmetric generalization of the original FSTWF potential.

The differential operators of D'Alambert and Dirac applied to the integral representations for the component fields allow to obtain the explicit form for the components (24) of the current multiplet $\mathcal{J}(x,\theta,\bar{\theta})$

$$j^{(2)}_\mu = e\int d\tau\Big[\dot{y}_\mu - \varepsilon_{\mu\nu\rho\lambda}\dot{y}^\nu(\xi\sigma^\rho\bar{\xi})\partial^\lambda + \frac{1}{4}\xi\xi\bar{\xi}\bar{\xi}\dot{y}^\nu(\partial_\mu\partial_\nu$$
$$-\eta_{\mu\nu}\Box) + \left(\xi\xi(\dot{\bar{\xi}}\tilde{\sigma}_{\mu\rho}\bar{\xi}) + (\dot{\xi}\sigma_{\mu\rho}\xi)\bar{\xi}\bar{\xi}\right)\partial^\rho\Big]\delta^{(4)}(s_0),$$
$$j^{(1)}_\alpha = e\int d\tau\Big[\dot{y}_\mu[(\sigma^\mu\bar{\xi})_\alpha - \frac{i}{2}\xi_\alpha\bar{\xi}\bar{\xi}\partial^\mu - i(\sigma^{\mu\rho}\xi)_\alpha\bar{\xi}\bar{\xi}\partial_\rho]$$
$$+i\dot{\xi}_\alpha\bar{\xi}\bar{\xi} - \frac{1}{2}(\sigma^\rho\dot{\bar{\xi}})_\alpha\xi\xi\bar{\xi}\bar{\xi}\partial_\rho\Big]\delta^{(4)}(s_0). \tag{30}$$

Now we choose another representation of kernel $\mathcal{K}$

$$\mathcal{K}^{(\mu)}_\alpha = \mu\left(\dot{\xi}_\alpha - i\dot{\bar{\xi}}^{\dot{\alpha}}\bar{\Delta}\bar{\Delta}\partial_{\alpha\dot{\alpha}}\right), \tag{31}$$

where the constant $\mu$ has the dimensionality of the length $L$ and has a physical meaning of the anomalous magnetic moment (AMM) of a neutral source particle [10]. This new representation satisfies the same conditions for the desired kernels as the above mentioned one. The obtained fields are generated by superparticles with nonzero AMM.

The coordinate integral representations of the main superfields and component fields can be written in the same manner as for the charged superparticle-source.

$$A^\alpha = \mu\int d\tau\left[\dot{\xi}^\alpha - i\dot{\bar{\xi}}^{\dot{\alpha}}\bar{\triangle}\bar{\triangle}\partial_{\alpha\dot{\alpha}}\right]\delta(s_R^2),$$
$$A_\nu = \mu\int d\tau\Big\{2\left[\dot{\xi}\sigma_{\mu\nu}\triangle - \bar{\triangle}\tilde{\sigma}_{\nu\mu}\dot{\bar{\xi}}\right]\partial^\mu +$$
$$+\frac{i}{2}\left[\triangle\triangle\dot{\xi}\sigma^\rho\bar{\triangle} + \bar{\triangle}\bar{\triangle}\triangle\sigma^\rho\dot{\bar{\xi}}\right](\eta_{\rho\nu}\Box - \partial_\rho\partial_\nu)\Big\}\delta(s^2),$$
$$W^\alpha = -i\mu\int d\tau\Big[\dot{\bar{\xi}}_{\dot{\alpha}}\partial^{\dot{\alpha}\alpha}\delta(s_L^2) - i\dot{\xi}^\alpha\Delta\Delta\Box\delta(s_R^2)\Big],$$
$$\Phi = -4\mu\int d\tau\left[\dot{\xi}^\alpha\Delta_\alpha\delta(s_R^2) + \dot{\bar{\xi}}_{\dot{\alpha}}\bar{\triangle}^{\dot{\alpha}}\delta(s_L^2)\right],$$
$$v_\nu = \mu\int d\tau\Big[-2i\left(\dot{\xi}\sigma_{\mu\nu}\xi - \bar{\xi}\tilde{\sigma}_{\nu\mu}\dot{\bar{\xi}}\right)\partial^\mu$$
$$+\frac{1}{2}\left(\xi\xi\dot{\xi}\sigma^\rho\bar{\xi} + \bar{\xi}\bar{\xi}\xi\sigma^\rho\dot{\bar{\xi}}\right)(\eta_{\rho\nu}\Box - \partial_\rho\partial_\nu)\Big]\delta(s_0^2), \tag{32}$$
$$\lambda^\alpha = \mu\int d\tau\left[-i\dot{\xi}^\alpha\xi\xi\Box + \dot{\bar{\xi}}_{\dot{\alpha}}\left(\partial^{\dot{\alpha}\alpha} + i\xi^\beta\bar{\xi}^{\dot{\beta}}\partial^{\dot{\alpha}\alpha}\partial_{\dot{\beta}\beta} + \frac{1}{4}\xi\xi\bar{\xi}\bar{\xi}\partial^{\dot{\alpha}\alpha}\Box\right)\right]\delta(s_0^2), \tag{33}$$

$$D = -\frac{1}{2}\mu \int d\tau \left[ 2(\dot{\xi}\xi + \dot{\bar{\xi}}\bar{\xi}) + i \left( \xi\xi\dot{\xi}\sigma^\mu\bar{\xi} - \bar{\xi}\bar{\xi}\xi\sigma^\mu\dot{\bar{\xi}} \right) \partial_\mu \right] \Box\delta(s_0^2). \tag{34}$$

$$j_\nu^{(2)} = \mu \int d\tau \Big[ -2i \left( \dot{\xi}\sigma_{\mu\nu}\xi - \bar{\xi}\tilde{\sigma}_{\nu\mu}\dot{\bar{\xi}} \right) \partial^\mu + \frac{1}{2} \left( \xi\xi\dot{\xi}\sigma^\rho\bar{\xi} + \bar{\xi}\bar{\xi}\xi\sigma^\rho\dot{\bar{\xi}} \right) (\eta_{\rho\nu}\Box - \partial_\rho\partial_\nu) \Big] \delta^{(4)}(s_0), \tag{35}$$

$$j_\alpha^{(1)} = \mu \int d\tau \left[ i\dot{\bar{\xi}}^{\dot{\alpha}}\bar{\xi}\bar{\xi}\partial_{\alpha\dot{\alpha}} - \dot{\xi}_\alpha \left( 1 - i\xi\partial\bar{\xi} + \frac{1}{4}\xi\xi\bar{\xi}\bar{\xi}\Box \right) \right] \delta^{(4)}(s_0). \tag{36}$$

The next important step of our work is the construction of the action functional for the interacting superparticles with charge and AMM. In the case of two interacting charged superparticles the corresponding action functional can be written as a sum of free and interaction part, which are defined by the expressions

$$S_0 = -\frac{1}{2}\int dt \Big( \frac{\omega_t^2}{g_t} + g_t m_1^2 \Big) - \frac{1}{2}\int d\tau \Big( \frac{\omega_\tau^2}{g_\tau} + g_\tau m_2^2 \Big),$$
$$S_{int}^{(e)} = i\,e \int dt \Big( \omega_t^\mu A_\mu + \dot{\theta}_t^\alpha A_\alpha + \dot{\bar{\theta}}_{t\dot{\alpha}} \bar{A}^{\dot{\alpha}} \Big) = ie \int \omega^M(d) A_M. \tag{37}$$

Here, instead of the gauge superfield $A_M$ we use their integral representations in the terms of the world coordinates $z^M$ and $\zeta^M$ of source particles without AMM. Starting the construction of the required here action note, that this functional will also remain invariant for the case when the superconnection $A_M$ is shifted on the SUSY- and $U(1)$-invariant object $W'_M = (W_\mu, W'_\alpha, \bar{W}'^{\dot{\alpha}})$. This object, in particular, may be constructed of the strength components $F_{MN}$. As is known [13], the inclusion of the interactions of spinning particles with external electromagnetic field by means of their AMMs requires using the superstrength components $F_{MN}$. So, it is reasonable to suppose that in the considered case an additional shift of the superconnection

$$eA_M \mapsto eA_M + i\mu_1 W'_M, \tag{38}$$

may be found sufficient for taking into account the interaction of superparticles with electromagnetic field via their AMM $\mu$. The realization of this assumption demands that the components of $W'_M$ should have proper dimensionalities $[W'_\mu] = L^{-2}$, $[W'_\alpha] = [\bar{W}'^{\dot{\alpha}}] = L^{-3/2}$. This circumstance, together with the requirement that $W'_M$ must be presented in the linear form with respect to the superfield invariant $F_{MN}$, sharply restricts the possibility to construct the invariants $W'_M$. In particular, the natural Lorentz vector $W'_\mu$ with the dimensionality $L^{-2}$ can not be constructed of $F_{MN}$. At the same time the desired spinor invariant $W'^\alpha$ may be taken in the form $\sim F_{\mu\dot{\alpha}}\tilde{\sigma}^{\mu\dot{\alpha}\alpha}$. So, the admissible shift may be chosen as

$$W'_M = W_M \equiv \frac{i}{4}(0, -\sigma_{\mu\alpha\dot{\alpha}}F^{\mu\dot{\alpha}}, \tilde{\sigma}^{\mu\dot{\alpha}\alpha}F_{\mu\alpha}). \tag{39}$$

In accordance with this choice, the action (37) for charged superparticles is generalized to the form

$$S_{int}^{(e,\mu)} = i \int \left[ \omega^\mu(d) e A_\mu + \theta^\alpha(d) \left( eA_\alpha + i\mu_1 W_\alpha \right) + \bar{\theta}_{\dot{\alpha}}(d) \left( e\bar{A}^{\dot{\alpha}} + i\mu_1 \bar{W}^{\dot{\alpha}} \right) \right]. \tag{40}$$

In general, $A_M$ and $W_\alpha$ in this expression, are the sum of two integral representations, which describe fields generated by charge and AMM of particle-source. It is not so difficult to verify that this action is symmetric one under permutations of the particles. Thus it solves the problem of the extension of the FSTWF approach for the one-half spin particles with electric charge and AMM. If we introduce the two-component "charge" $q^\Lambda \equiv (e, i\mu)$ and superconnection $G^\Lambda_M \equiv \begin{pmatrix} A_M \\ W_M \end{pmatrix}$ this action can be rewritten more compactly

$$S^{(e,\mu)}_{int} = i \int \omega^M(d) q^\Lambda G^\Lambda_M. \tag{41}$$

Such an "isotopic" form of notation allows to underline the symmetry between the pair: $(e, A_M) \leftrightarrow (i\mu, W_M)$. The considered modification of the superfield connection and the action leads to the change of the standard supersymmetric and $U(1)$-covariant derivative $\nabla_M$ into the new $\tilde{\nabla}_M$

$$\nabla_M \equiv D_M + eA_M \;\mapsto\; \tilde{\nabla}_M = D_M + q^\Lambda G^\Lambda_M = D_M + eA_M + i\mu W_M. \tag{42}$$

The corresponding change of the standard superfield strengths $eF_{MN}$ [9] into the extended ones $q^\Lambda G^\Lambda_{MN}$ is performed following way

$$q^\Lambda G^\Lambda_{MN} = eF_{MN} + 2i\mu D_{[M} W_{N\}} \tag{43}$$

and permits to rewrite equations of motion of superparticle with charge and AMM more compactly too.

The physical meaning of the constant $\mu$ as the anomalous magnetic moment of the particles follows from an analysis of action. After the first pair of Maxwell equations is taken into account, this action becomes

$$\begin{aligned} S^{(e,\mu)}_{int}\Big|_{\substack{e=0,\\ \text{photon}}} &= i\mu \int d\eta \left[(\dot{\theta}\sigma^{\mu\nu}\theta) - (\dot{\bar{\theta}}\tilde{\sigma}^{\mu\nu}\bar{\theta})\right] v_{\mu\nu} \\ &+ \frac{1}{2}\mu \int d\eta \left[\theta\theta(\dot{\theta}\sigma^\mu\bar{\theta}) + \bar{\theta}\bar{\theta}(\theta\sigma^\mu\dot{\bar{\theta}})\right] \partial^\rho v_{\rho\mu}. \end{aligned} \tag{44}$$

Of the two terms remaining in this action, the second describes the spin-orbit and other relativistic interactions, which correspond to succeding terms in the expansion in power of $1/c$. Accordingly, we can see the physical meaning of the constant $\mu$ by restricting the analysis to the first term. Here it is convenient to switch from the pair of Weyl spinors $(\theta, \bar{\theta})$ and the matrix $(\sigma^{\mu\nu})_\alpha^\beta$ to the Dirac bispinor $\Psi$ and the spin operator of the relativistic particle $\Sigma_{\mu\nu}$

$$\Psi = \begin{pmatrix} \theta_\alpha \\ \dot{\bar{\theta}}^{\dot{\alpha}} \end{pmatrix}, \qquad \Sigma_{\mu\nu} = \frac{i}{4}[\gamma_\mu, \gamma_\nu]. \tag{45}$$

Here $\gamma_\mu$ are the Dirac matrices in Weyl basis. The contribution of the first term to the action can then be written as a standard Pauli term

$$\begin{aligned} S^{(e,\mu)}_{int} = \mu \int d\tau \left(\bar{\psi}\Sigma^{\mu\nu}\psi\right) v_{\mu\nu} + \\ + (\text{high-order corrections and other interactions}) \end{aligned} \tag{46}$$

The physical meaning of the constant $\mu$ is obvious from this expression: it is the anomalous magnetic moment of the particle, expressed in Bohr magnetons.

Shown is the principal possibility of the unification of the action-at-a-distance theory together with the conception of supersymmetry. This unification permits to generalize the idea of the construction of fields from world coordinates for spinor fields. As a result, the fundamental Maxwell and Dirac equations are derived for the fields of the Maxwell supermultiplet. Another important result of the approach developed here is the generalization of the minimality principle taking into account the electromagnetic interaction via the anomalous magnetic moment of superparticle. This new form of the minimality principle gives a new universality relation between the pairs $(e, A_M)$ and $(i\mu, W_M)$. The significance of this extended minimality principle falls outside the frame of the action-at-a-distance theory and may be used as a general principle in the (super)gauge field theories.

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